



HISTORY OF ANALYTIC PHILOSOPHY



ALFRED TARSKI PHILOSOPHY OF LANGUAGE AND LOGIC

Douglas Patterson



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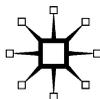
NEW ESSAYS ON TARSKI AND PHILOSOPHY (*editor*)

Alfred Tarski: Philosophy of Language and Logic

Douglas Patterson

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Series Editor's Foreword

During the first half of the twentieth century analytic philosophy gradually established itself as the dominant tradition in the English-speaking world, and over the last few decades it has taken firm root in many other parts of the world. There has been increasing debate over just what 'analytic philosophy' means, as the movement has ramified into the complex tradition that we know today, but the influence of the concerns, ideas and methods of early analytic philosophy on contemporary thought is indisputable. All this has led to greater self-consciousness among analytic philosophers about the nature and origins of their tradition, and scholarly interest in its historical development and philosophical foundations has blossomed in recent years. The result is that history of analytic philosophy is now recognized as a major field of philosophy in its own right.

The main aim of the series in which the present book appears – the first series of its kind – is to create a venue for work on the history of analytic philosophy, consolidating the area as a major field of philosophy and promoting further research and debate. The 'history of analytic philosophy' is understood broadly, as covering the period from the last three decades of the nineteenth century to the start of the twenty-first century – beginning with the work of Frege, Russell, Moore and Wittgenstein, who are generally regarded as its main founders, and the influences upon them – and going right up to the most recent developments. In allowing the 'history' to extend to the present, the aim is to encourage engagement with contemporary debates in philosophy – for example, in showing how the concerns of early analytic philosophy relate to current concerns. In focusing on analytic philosophy, the aim is not to exclude comparisons with other – earlier or contemporary – traditions, or consideration of figures or themes that some might regard as marginal to the analytic tradition but which also throw light on analytic philosophy. Indeed, a further aim of the series is to deepen our understanding of the broader context in which analytic philosophy developed, by looking, for example, at the roots of analytic philosophy in neo-Kantianism or British idealism, or the connections between analytic philosophy and phenomenology, or discussing the work of philosophers who were important in the development of analytic philosophy but who are now often forgotten.

In this book Douglas Patterson provides the first full-length account of Alfred Tarski's philosophy. Tarski was born in Warsaw in 1901 and gained his doctorate in logic at the University of Warsaw in 1924, supervised by Stanislaw Leśniewski. In the 1920s and 1930s he published extensively on logic and set theory, and as a representative of the so-called Lvov–Warsaw School, maintained close links with Gödel, Carnap and other members of the Vienna Circle. When Nazi Germany invaded Poland on 1 September 1939, Tarski was at a conference in the United States and was unable to return home. He stayed there throughout the war years, teaching at Harvard, New York and Princeton before eventually being given a permanent post at the University of California at Berkeley in 1945, where he remained until his death in 1983. Tarski was thus one of the many logicians and philosophers from Central Europe who moved to the United States as a result of the rise of Nazism in Germany, and whose story is part of the broader story of the development of analytic philosophy in North America, as the ideas of the Polish logicians and the logical positivists took root in new soil.

Tarski's two most famous papers are 'The Concept of Truth in Formalized Languages', first published in Polish in 1933 (and in German in 1935) and 'On the Concept of Logical Consequence', published in both Polish and German in 1936. In the first paper Tarski offers a definition of truth for formal languages by introducing the notion of satisfaction and appealing to the recursive structure of formal languages. Tarski first formulates his 'T-schema' here (famously exemplified in the statement that 'Snow is white is true' if and only if snow is white), and also offers a solution to the Liar paradox by insisting that truth for a language can only be defined in a metalanguage. In the second paper Tarski offers a corresponding semantic definition of logical consequence by utilizing the idea of truth-under-an-interpretation introduced in his earlier paper. These two papers have been seen as establishing the foundations of truth-conditional semantics, with Tarski also regarded as a key figure in the development of model theory.

As Patterson argues in this book, however, Tarski's actual views are both more complex and more intriguing than they have standardly been taken to be. Patterson begins by distinguishing what he calls the 'expressive' conception of meaning from the 'representational' conception of meaning. On the expressive conception, language expresses thoughts and the notions of assertion and justification have primacy. On the representational conception, language represents things in the world and the notions of reference and truth have primacy. The former finds its natural home in the proof-theoretic conception of logic, while the latter

is reflected in the model-theoretic conception of logic. Patterson agrees that Tarski's two papers made major contributions to representational semantics and model theory, but by carefully examining the development of Tarski's work from the late 1920s to the mid-1930s, he shows that that work was originally motivated by the expressive conception of meaning. Tarski himself referred to his earlier view as 'intuitionistic formalism', a view that he inherited from Leśniewski. It was only once he had convinced himself that logical consequence could be defined semantically that he abandoned his earlier view. Patterson remarks in his introduction that "Tarski may well be the most enigmatic figure in the history of analytic philosophy". Patterson's detailed account of the crucial period in Tarski's intellectual development not only sheds a great deal of light on Tarski's evolving ideas on the fundamental notions of logic and semantics but also inaugurates a new era in our understanding of Tarski's work and its contribution to analytic philosophy.

Michael Beaney
September 2011

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Introduction

0.1 Expressive and representational semantics

This book tells the story of the birth of truth-conditional semantics from an earlier conception of meaning. If we think of language as standing between mind and world, there are two simple ways in which to think of it as meaningful: in terms of its relation to the mind, or in terms of its relation to the world. One may thus conceive of meaning in terms of the expression of thought, or in terms of the representation of things. Call the two conceptions *expressive* and *representational* semantics. On the expressive conception the function of language is to express thoughts, which may themselves be representational. On the representational conception language is conceived of as representational in its own right, and the expression of thought is a derivative function. Central to the expressive conception of language are the notions of assertion and justification since our basic notions are a subject's saying something and their reasons for doing so; central to the representational conception are the notions of reference and truth being about things and accurately representing how they are.

Both views address the topic of inference. The expressive conception offers a natural, intuitive connection to the basic idea of an argument as something that gives one a reason to believe one thing given that one believes others: in a valid argument, justification for believing the premises is transmitted to the conclusion and becomes justification for believing it. The representational conception, since its focus is on accuracy in what statements are about, gives rise to the idea that an argument is valid just in case if its premises are true, so is its conclusion. Continuing in the same broad brush-strokes, the expressive conception sits naturally with the conception of inference as derivation of one claim from others

in accord with intuitively valid rules—a conception that becomes, in more refined studies of logic, proof theory. The representational conception naturally leads to the idea of an inference as valid just in case all models of the premises are also models of the conclusion, an idea at the foundation of model-theoretic studies of logic. The interaction of the two conceptions in their refined forms then gives us two of the central results of the 20th century, Gödel's completeness and incompleteness theorems.

Neither conception of meaning is sufficient of itself; likewise, neither the proof-theoretic nor the model-theoretic conception of logic of itself captures the notion of one thing's following from another. Indeed, when it comes to logic, both derivation and semantic consequence are arguably incomplete or even inaccurate representations of what it is for one claim to follow from others. Derivational conceptions of consequence seem limited to the finite, and there isn't anything particularly explicit in the "therefore" of ordinary deductive argumentation that concerns rules of proof, while semantic conceptions of consequence seem to lose the epistemic cast of this same "therefore", since one thing might be the case if others were even if their being so gave one, even in principle, no reason at all to think that it was so. The relations among the intuitive notions of deductive validity, of derivation, and of semantic consequence will be a matter for further investigation as we go along. However, whatever the consequences of that investigation might be, the refinement of these two traditions into proof theory and model theory was a major achievement.

Alfred Tarski is remembered by students of language for contributions that made rigorous treatments of the central concepts of the second of these two traditions possible. In "The Concept of Truth in Formalized Languages" (CTFL) and related papers in the early 1930s Tarski developed the tools and techniques that are still at the center of logic, the philosophy of language and, to a lesser extent, linguistic semantics. Indeed, Tarski's achievements in this regard were so influential that today it is forgotten that representational semantics as he developed it was devised as a contribution to a certain project motivated by the expressive conception. Standard reports on Tarski's work simply treat it as an obvious early contribution to representational semantics. Lurking in the text of the classic papers, however, is a different project, one to which representational semantics was at first intended as a small contribution. Little is known about this project today other than that at one place [Tarski, 1983a, 62] Tarski refers to the view he was working with as "Intuitionistic Formalism".

In this work we will examine the development of truth-conditional, representational semantics by Tarski from the late 1920s until the mid-1930s. Three basic foci of attention will allow us to fill in the surrounding picture from which representational semantics is usually anachronistically abstracted. First, we will examine what Intuitionistic Formalism was, where it came from, and how Tarski conceived of himself as contributing to it. Second, we will look at the development of semantics from its beginnings within Intuitionistic Formalism to its replacement of it in 1935. Finally, we will trace Tarski's treatment of logical consequence through its development in his thinking, for it was when Tarski realized that semantics as he had developed it could replace the conception of inference required by Intuitionistic Formalism that the latter view itself lost its hold on him.

Our story, summarized, goes like this. Tarski originally became interested in the question of how our thoughts and ideas, under a certain conception of what those are, could adequately be expressed in an axiomatic theory or "deductive science", a topic that was central to his advisor Stanislaw Leśniewski's work. As Intuitionistic Formalism was conceived of by Leśniewski as a conception of the function and significance of an axiomatic theory, Tarski set himself the task of exploring how the basic concepts used in thought *about* axiomatic theories—consequence, truth, reference and related notions such as completeness and categoricity (in several senses)—could themselves be captured, to Intuitionistic Formalist standards, within an axiomatic theory. In particular due to his interest in early work in what we would now think of as model theory, Tarski set out to develop a way of capturing the semantic notions of truth, satisfaction and reference within such a theory to Intuitionistic Formalist standards. The result of this project was the now-familiar method of defining truth by recursion on satisfaction.

Logical consequence, on the other hand, was much slower to develop in Tarski's hands. Aside from a skeptical footnote following Gödel [Tarski, 1983a, 252], as late as 1934 Tarski treated logical consequence derivationally, in terms of a recognized set of apparently valid rules for asserting sentences given that others had been asserted. Only when he realized that the notion of a model could be defined in terms of semantics, and consequence in turn defined in terms of that, did Tarski see that semantics could actually stand on its own as a treatment of language, and at that point he moved on from the project that had originally motivated him: after 1936 Intuitionistic Formalist concerns disappear from his work and, in particular, 1944's "The Semantic Conception of Truth", though

it appears to summarize the work of the 1930s, leaves out the themes characteristic of Intuitionistic Formalism.

0.2 The received view

Tarski may well be the most enigmatic figure in the history of analytic philosophy. Although his importance is widely accepted, his work is rarely actually read and interpretations and assessments of his views differ wildly. It was this way from the beginning. Popper first learned of Tarski's innovations when he asked Tarski to explain his work:

and he did so in a lecture of perhaps twenty minutes on a bench (an unforgotten bench) in the *Volksgarten* in Vienna. He also allowed me to see the sequence of proof sheets of the German translation of his great paper on the concept of truth, which were then just being sent to him ... No words can describe how much I learned from all this, and no words can express my gratitude for it. Although Tarski was only a little older than I, and although we were, in those days, on terms of considerable intimacy, I looked upon him as the one man whom I could truly regard as my teacher in philosophy. I have never learned so much from anybody else [Popper, 1974, 399].

Carnap recalls his own initiation:

When Tarski told me for the first time that he had constructed a definition of truth, I assumed that he had in mind a syntactical definition of logical truth or provability. I was surprised when he said that he meant truth in the customary sense, including contingent factual truth. Since I was only thinking in terms of a syntactical metalanguage, I wondered how it was possible to state the truth-condition for a simple sentence like "this table is black". Tarski replied: "This is simple: the sentence 'this table is black' is true if and only if this table is black" ... When I met Tarski again in Vienna I urged him to deliver a paper on semantics and on his definition of truth at the International Congress for Scientific Philosophy to be held in Paris in September. I told him that all those interested in scientific philosophy and the analysis of language would welcome this new instrument with enthusiasm, and would be eager to apply it in their own philosophical work. [Carnap, 1963, 60–1].

However, Tarski was skeptical, and rightly so. On the one hand, that "this table is black" is true if and only if this table is black can easily seem something less than a penetrating insight; on the other hand, as

Carnap's reference to syntax indicates, there was at the time a great deal of skepticism or outright hostility toward the treatment of language in terms of word–world relations. The latter reaction dominated the the Congress, where “there was vehement opposition even on the side of our philosophical friends” [Carnap, 1963, 61]. Posterity hasn't been more accommodating, though the sources of resistance have shifted. Consider these familiar assessments of Tarski's views:

The concern of philosophy is precisely to discover what the intuitive notion of truth is. As a philosophical account of truth, Tarski's theory fails as badly as it is possible for an account to fail [Putnam, 1994, 333].

My claim is that Tarski's analysis is wrong, that his account of logical truth and logical consequence does not capture, or even come close to capturing, any pretheoretic conception of its logical properties [Etchemendy, 1990, 6].

What provoked these disparate reactions? The standard conception of Tarski's views is relatively easy to summarize. In “The Concept of Truth in Formalized Languages” [Tarski, 1983a] (CTFL) Tarski sets himself the task of “defining truth” and maintains that a good definition of truth would be something that, added to the formal syntax of a language, results in implication of a sentence of the form “*s* is true if and only if *p*” (as we now say, a “T-sentence”) for each sentence of the language, where what is substituted for “*p*” is or translates *s*. If this is the standard, Tarski notes, then if a language had a finite number of sentences one could simply list the T-sentences and be done with it [Tarski, 1983a, 188]. The technical achievement comes in Tarski's recognition that if a language has an infinite number of sentences and one demands a finite number of axioms in one's syntactic-cum-semantic theory, some use of “the recursive method” [Tarski, 1983a, 189] is required. Furthermore, since complex sentences aren't necessarily built out of sentences (e.g. $\exists xFx$ is built by concatenating the quantifier with the open “*Fx*”) the basic recursive definition has to concern something other than closed sentences, but then produce, for closed sentences, a definition with the T-sentences as consequences.

The details here are familiar and so I refer the reader to standard presentations such as [Soames, 1999]. The basics are encountered in any advanced undergraduate logic class. Consider a language \mathcal{L} , with names, predicates and functors and variables of the first and perhaps higher orders and quantifiers to bind them, plus some stock of sentential connectives. Syntactically, one sets out rules for forming a sentence

(open or closed) by stringing together names, functors, predicates, variables, quantifiers and connectives; some notational bookkeeping is done with devices like parentheses, the dots of the Peano school, or order alone, as in Polish notation, to determine things like the scope of a quantifier or the main connective of a sentence. The rules sort strings into formulas and non-formulas. Since they are recursive (e.g. one can conjoin a conjunction with something else or apply a functor to something that contains functors) the result is an infinite set of formulae, some open and some closed, where an open formula contains at least one variable bound by no quantifier.

Semantically, by Tarski's techniques, one assigns denotations to names and sets of ordered n -tuples to predicates and to functors. The former n -tuples represent objects as standing in the relation expressed by the predicate and the latter have a distinguished element (the last, say) that expresses the reference of the functor when it is applied to the first $n - 1$ members of the n -tuple as arguments. Truth-functions are assigned to the sentential connectives in the obvious way. Variables are enumerated and objects in the domain of discourse are taken to form sequences—enumerations of objects to correspond to enumerations of variables. A sentence s with a relation symbol R and no bound variables is satisfied by a sequence just in case the n -tuple of objects with indices that match those of the free variables in s is one of the n -tuples assigned to R . If the sentence involves a universal quantifier binding variable x_i it is satisfied by a sequence if and only if every sentence that differs from that sequence at most the x_i^{th} place satisfies the open sentence from which it was formed by adding the quantifier [Tarski, 1983a, 193]. Closed sentences being those with no free variables, they are satisfied by every sequence if they are satisfied by any, and so a true sentence is defined as one satisfied by every sequence [Tarski, 1983a, 194–5]. Given that reference and satisfaction have themselves been introduced by translational enumeration (e.g. “Frankreich’ refers in German to France”) the result is an account that implies the T-sentences as desired.

As the standard conception notes, Tarski was motivated by the anti-nomy of the liar and related semantic paradoxes and took them to be the primary obstacle to a rigorous treatment of the concept of truth. These lead him to say that no language can contain a predicate applying to exactly the set of (Gödel codes of) its own truths and that therefore the definition of truth for a language can only be given in a “richer” metalanguage. The result, on the standard reading, is the “Tarski hierarchy” in which the semantics for a language must be given in metalanguage that cannot be translated into it, the semantics of which in turn must

be stated in a yet higher “meta-metalanguage”, and so on. Since natural languages appear to contain self-applying semantic vocabulary and this apparently violates the restrictions imposed by the hierarchy, Tarski condemns them as “inconsistent” and rejects them as unfit for rigorous study.

Continuing with the standard narrative, in 1935–6 Tarski turned his attention to the relation of logical consequence. Here he defines a model of a set of sentences as a sequence that satisfies all of them and then defines an argument (Γ, s) as valid just in case every model of Γ is a model of s . There are some oddities about Tarski’s presentation, but it is widely held to be more or less the first statement of the usual contemporary conception of model-theoretic consequence.

As we have noted, reactions have been extreme. Neurath thought that Tarski was rehabilitating metaphysics and rejected the idea that the truth of a sentence could be understood in terms of worldly conditions rather than its confirmation within a system of scientific statements [Mancosu, 2008]. Later thinkers have questioned whether Tarski really provided a “reduction” of semantic notions to physicalistically acceptable terms [Field, 1972]. Interpreters have often objected to the apparent restriction of Tarski’s “definition” of truth to a single language. Others have pilloried Tarski’s remarks on natural language. Tarski’s response to the paradoxes, more generally, is the main foil for contemporary work on the paradoxes, work that attempts to show that, contrary to his pronouncements, languages that contain their own truth-predicates can be constructed. As for Tarski’s account of consequence, though it has found defenders, many agree with Etchemendy’s assessment quoted above.

0.3 Themes

So much for the received view. For those keeping score, I will mostly disagree with the usual objections to Tarski’s views on truth and paradox and mostly agree with the objections to Tarski’s account of consequence. The details will emerge as we progress. But keep in mind that my goal here is to explain where Tarski’s views came from and how they inform what he wrote at the time rather than to defend everything that he said.

The overall story here is a developmental one. Tarski begins in the late 1920s working within a conception of language and logic inherited from his mentors Stanislaw Leśniewski and Tadeusz Kotarbiński. He sets himself the task of working out “Intuitionistic Formalist” accounts of basic metatheoretical notions—in the first case, the consequence relation conceived of as determined by a set of primitively accepted rules of

inference [Tarski, 1983c, 63]. Sometime in 1929 he turns his attention to semantic notions and develops the basic techniques described above, at first imperfectly in [Tarski, 1983f] and then in full form in [Tarski, 1983a]. During this period he doesn't think to apply the techniques to the consequence relation, nor does he notice other obvious applications, e.g. to Padoa's method for proving the independence of a set of axioms. These omissions are important because they show the extent to which Tarski is still thinking squarely in terms of Intuitionistic Formalism. Sometime in 1934–5 Tarski realized that he could replace the appeal to primitively valid inference rules with a notion of consequence defined in terms of his semantics. The result is the account of [Tarski, 2002] and with that the project came to an end.

As a bit of a preview, here are some themes that we will be exploring as we move along.

1. *Intuitionistic Formalism*: What was it and how did it inform Tarski's work from 1926–1936?
2. *Definition*: What was Tarski's conception of definition? How do his definitions relate to what philosophers offer when they "analyze" concepts?
3. *Language*: What were Tarski's views on Language? What was the implicit account of meaning and communication with which Tarski worked?
4. *Semantics*: What motivated Tarski's project of defining semantic terms in mathematical theories? How did he understand what he was doing at the time?
5. *Compositionality*: Intuitionistic Formalism not only lacked any articulate account of how the meanings of a complex expression were determined by those of its parts—something required of any reasonable treatment of language—but in Tarski's hands it involved an account of meaning that was incompatible with compositionality. We will see how Tarski's semantics begins to supply this. One of the aspects of this shift is the move from a "holist" account of meaning on which theories determined the meaning of their sentences and expressions, to an "atomist" one which the determination runs from lexical expressions to sentences.
6. *Consequence*: Intuitionistic Formalism involved a conception of logical consequence on which it was a matter of primitively valid rules of inference. Tarski's successor conception detaches consequence from this requirement. The result is that Tarski moves from a conception of consequence that places its epistemic aspects at the forefront to

- one that stresses the independence of logic from the semantics of non-logical terms. Call the contrast one between “epistemic” and “generality” conceptions of consequence. Whether this is laudable is something we will consider at length.
7. *Logical Monism and Pluralism*: Tarski began his work accepting that Simple Type Theory (STT), conceived of in terms of the “theory of semantical categories” he attributed to Leśniewski and others, was logic. Under the influence of Carnap in particular, but also paying attention to other developments, he moved to a more pluralist or, as Carnap would put it, “tolerant” attitude.
 8. *The Status of Mathematics*: Tarski inherited a liberal Polish attitude about the status of mathematics. Though he nowhere takes the logicist program as a motivation, a good deal of his views assume that mathematics, to the extent that it can be developed in STT, is logic. Later, especially beginning with [Tarski, 2002] a more open-minded attitude about logic and mathematics came to the fore.

Those, then, are some of the themes that we will explore. That said, I owe the reader some words about what will not be covered in this book. Our topic is Tarski’s evolving set of views about logic and language, and in particular “formal axiomatics” in the period 1926–1936. Tarski’s mathematical work from the period will not be covered.¹ Nor will I have space to discuss Tarski’s views after 1936, with the exception of “The Semantic Conception of Truth”, published in 1944 but largely a response to the debate that arose following his presentation of “The Establishment of Scientific Semantics” (ESS) and “On the Concept of Following Logically” (CFL) at the Paris Unity of Science Congress in 1935.²

As for other sources, in order to stay within the publisher’s length limit I have set myself the basic rule of sticking to figures and works Tarski himself mentions, with only a few exceptions when these seemed especially pertinent. Missing, in particular, are the discussions of Russell and Frege that one might expect in a work on the history of analytic philosophy. Tarski was in a position to have received at least some Frege from Leśniewski, but Frege played no direct role in his thought. Russell seems relevant at the many places where Tarski mentions *Principia Mathematica*, but since Tarski mentions it only to set it aside in favor of STT, I found little of use in bringing Russell into our discussion. Wittgenstein will figure into our story only indirectly, as filtered through Carnap and the Vienna Circle. One comparison seemed too interesting to me to leave out, and so it occurs in footnotes: that of Tarski’s work with contemporaneous writings of his unfortunately neglected colleague Kazimerz Ajdukiewicz.